Colour chiral solitons in low energy QCD

Victor Novozhilov*and Yuri Novozhilov†
V.Fock Department of Theoretical Physics,
St.Petersburg State University, 198504, St.Petersburg, Russia

Abstract

We derive an effective colour chiral action with a background gauge field. The action describes configurations of soliton-skyrmion type. Kinetic term constant f_0^2 , analogue of f_π^2 , is a phenomenological dimensional parameter of the model; $d = 4$ terms are unique up to the choice of background gauge field. The case of SU(2) colour group is discussed in detail. We study an isolated configuration, i.e. a configuration in a background field which is the vacuum field forming the gluon condensate. Thereby we introduce the condensate energy as a second parameter and scale. Compared with the case of flavor skyrmion configuration, the colour chiral action contains a piece with slowly decreasing terms coming from the background vacuum field. Asymptotic behaviour at large distances shows exponential decrease for the case of chromomagnetic condensates and periodic otherwise . This defines a stability region for a colour soliton. The mass is given by the positive definite integrand for a soliton of purely bosonization origin. Contribution from the Yang-Mills action of the background colour field has the sign opposite to bosonization part. The baryon number current is not influenced by the background field and leads to the standard baryon number $B = N_F/N_C$. For $B = 1/3$ we evaluated the estimate from above $M \approx 460\ Mev$.

Keywords: [colour chiral model, colour soliton]

0.1 Introduction

In low energy QCD the ideas of Skyrmions [1, 2, 3, 4] and gauged solitons [5, 6] showed themself fruitful. To find stable colour configurations can be an important next step towards understanding of diquarks and exotic hadrons. A possibility of colour chiral solitons with baryon number N_F/N_C was mentioned in the first paper on colour bosonization [7] . However, the effective action in bosonization [7] was implicitly gauge dependent, the choice of background colour fields was not discussed, and soliton stability was not

*E-mail : vnovozhilov@mail.ru
†E-mail : yunovo@pobox.spbu.ru

investigated. It was found [8] that direct application of an effective bosonized lagrangian [7] does not lead to stable configurations. The idea of colour skyrmions from different viewpoints was explored [9, 10, 11, 12] in attempt to construct a constituent quark for $N_F = 1$, but further development in this direction was suspended after the conclusion [13] that stable colour solitons do not exist.

The aim of this paper is to study a gauge invariant effective chiral lagrangian and investigate stability of soliton configurations in the gluonic vacuum.

The chiral colour bosonization in QCD follows , in general, the lines of flavor bosonization [14, 15, 16, 17] . However, the vector colour (gluonic) field is a dynamical gauge field, while the flavor field in the Dirac lagrangian is an external one. In order to get a chiral colour action, the background field should be also chirally rotated giving an additional contribution to the standard chiral action. In flavor bosonization no such terms are present.

For a colour soliton, the background field describes soliton environment and produces corresponding interaction terms in the effective chiral lagrangian. In this paper we consider a separate (free) soliton, and, therefore, take colour vacuum field as a background field. Such colour vacuum fields form the gluon condensate. Experimentally, the condensate is positive, so that the vacuum field is chromomagnetic. Its value will be a parameter of the theory. The vacuum field gives rise to terms in the effective chiral lagrangian with $R^2 \sin^2 F$ and $R^4 \sin^4 F$ where R is a distance from the center of the standard hedgehog configuration of the shape F. . Asymptotic behaviour of soliton configuration at $R \to \infty$ is determined by the condensate: it is exponentially decreasing for positive condensate and periodic otherwise.

In section 2 we derive a general expression for an effective colour chiral action starting from the Dirac lagrangian with N_C colours and N_F flavors. In section 3 we make choice of a background field as a vacuum colour field which forms the gluon condensate, work out an effective chiral action in the case of the gauge group SU(2) for the hedghog configuration, study asymptotic behavior and evaluate the mass.

0.2 Colour bosonization and effective action

Bosonization is a standard prescription for introducing chiral field, and integration of chiral anomaly is usually invoked, as a way to the chiral action. In the case of the colour chiral field the Dirac lagrangian is not chiral invariant because of quark-gluon interaction term, so that there is no anomaly. However, what is really important in order to get the chiral action, is non-invariance of the fermion vacuum functional under chiral transformations, while the classical lagrangian does not need to be invariant. Such a non-invariance is related to the fact that the chiral field belongs to fermionic degrees of freedom (chiral phase) present in the functional measure, and an effective action arises from the Jacobian of the transformation to new variables including colour chiral phase. We present briefly the way to an effective action from this point of view [18].

To illustrate the approach we consider a field $\Phi(x)$ with the Lagrangian $L(x)$ and vacuum functional

$$Z = \int D\Phi e^{i\int Ldx} \tag{1}$$

Let $\Pi(x)$ be a local field with quantum numbers of a composite system. made out of Φ .The functional measure $D\Phi$ is over all independent degrees of freedom of Φ; thus it includes also degrees of freedom of $\Pi(x)$ together with remaining degrees X which are considered inessential. If we change variables

$$\{\Phi\} \to \{\Pi, X\}, \tag{2}$$

and transform Z

$$Z = \int D\Pi DXJ\left(\partial\Phi/\partial\Pi, \partial\Phi/\partial X\right) e^{i\int L(\Pi,X)dx} \equiv Z_{inv} \int D\Pi e^{i\int L_{eff}(\Pi)dx} \tag{3}$$

then after integrating out inessential variables X we arrive at the effective Lagrangian $L_{eff}(\Pi)$ for a collective variable Π describing a composite field. The functional Z_{inv} does not depend on Π ; J is a Jacobian of transformation $\{\Phi\} \to \{\Pi, X\}$.

In practice, to find directly variables X and the Jacobian is a difficult task. However, there are some classes of collective variables Π, when knowledge of X is not necessary in order to find an effective Lagrangian $L_{eff}\left(\Pi\right)$, namely, when Π are parameters of non-invariance groups and different classes correspond to different groups. "Non-invariance" is understood in relation to the vacuum functional : $\delta Z/\delta\Pi \neq 0$.

Consider a group H of transformations $U = \exp i\Pi$ of field Φ, $\Phi \to \Phi^U, U_3 = U_2U_1$, with the invariant measure $D(U'U) = DU$ and the vacuum functional

$$Z\left[U\right] = \int D\Phi \exp\left(i\int dxL\left(\Phi^U\right)\right) \tag{4}$$

Integrating over U we get H - invariants

$$Z_0 = \int DUZ\left[U\right], Z_{inv}^{-1} = \int DUZ^{-1}\left[U\right] \tag{5}$$

which can be used in order to substract from Z a H-invariant part leaving a functional Z_U for U

$$ZZ_0^{-1} \simeq ZZ_{inv}^{-1} = Z_U \equiv \int DU \exp\left(i\int dxL_{eff}\left(U\right)\right) \tag{6}$$

and identifying an effective action for U as

$$W_{eff}\left(U\right) = \int dxL_{eff}\left(U\right) = -i\ln\frac{Z}{Z\left[U\right]} \tag{7}$$

We use $\simeq$ to show that Z_0 and Z_{inv} differ by H -invariant terms. Thus, L_{eff} is defined up to U -independent terms.

We see that inessential variables X were effectively integrated out in integration over all initial degrees of freedom. A replacement $\Phi \to \Phi^U$ in both the measure and the lagrangian in Z is just a change of notations and cannot change Z. Thus , the Jacobian J in (3) is expressed in terms of $W_{eff}\left(U\right)$.

Let us consider the Dirac lagrangian $L_{\psi}(G,A)$with background gluon field G_{μ} and external colour field A_{μ} in the group SU(N)$\times SU(N)$

$$L_{\psi}=i\overline{\psi}\left(\widehat{\partial}+\widehat{G}+\gamma_{5}\widehat{A}\right)\psi=\overline{\psi}D\left(G,A\right)\psi, \tag{8}$$

A chiral field U is defined by the following transformation of Dirac fermions

$$\psi^{U}=\frac{1}{2}\left[\left(1-\gamma_{5}\right)U+\left(1+\gamma_{5}\right)\right]\psi, U^{+}U=1 \tag{9}$$

The quark Lagrangian $L_{\psi}\left(G,A\right)$ remains invariant, if fields G_{μ},A_{μ} are transformed appropriately

$$L_{\psi}\left(G,A\right)=\overline{\psi^{U}}D\left(G^{U},A^{U}\right)\psi^{U} \tag{10}$$

where U -transformed fields are given by

$$G_{\mu}^{U}+A_{\mu}^{U}=U\left(G_{\mu}+A_{\mu}\right)U^{-1}+U\partial_{\mu}U^{-1},G_{\mu}^{U}-A_{\mu}^{U}=G_{\mu}-A_{\mu} \tag{11}$$

Repeated transformation with the chiral field U_{1} gives $(G_{\mu}^{U})^{U_{1}}=G_{\mu}^{U_{1}U}$. These fields are not symmetrical /antisymmetrical with respect to left-right exchange. However, they are gauge transforms of vector and axial vector fields $\widetilde{G_{\mu}},\widetilde{A}_{\mu}$ with the gauge function χ which is square root of U

$$G_{\mu}^{U}=\chi\widetilde{G}_{\mu}\chi^{-1}+\chi\partial_{\mu}\chi^{-1},A_{\mu}^{U}=\chi\widetilde{A}_{\mu}\chi^{-1},\chi^{2}=U \tag{12}$$

Under a gauge transformation g the function χ transforms as $\chi'=g\chi g^{-1}$.

The infinitesimal chiral transformation $g_{5}=1+\gamma_{5}\lambda$ acts in the following way

$$\delta G_{\mu}=\left[A_{\mu},\lambda\right],\delta A_{\mu}=D_{\mu}\lambda$$

$$\delta\psi=\gamma_{5}\lambda\psi,\delta\overline{\psi}=\overline{\psi}\gamma_{5}\lambda,\delta U=-\left(U\lambda+\lambda U\right) \tag{13}$$

An important property of U -transformed fields $G^{U},A^{U},\psi^{U},\overline{\psi^{U}}$is that for them the chiral transformation g_{5} is a non-chiral gauge transformation

$$\delta G_{\mu}^{U}=-D_{\mu}\left(G^{U}\right)\lambda,\delta A_{\mu}^{U}=\left[\lambda,A_{\mu}^{U}\right]$$

$$\delta\psi^{U}=\lambda\psi^{U},\delta\overline{\psi}^{U}=-\overline{\psi}^{U}\lambda \tag{14}$$

It follows that the Yang-Mills Lagrangian $L_{YM}\left(G^{U}\right)=\frac{1}{2g^{2}}\mathrm{tr}G_{\mu\nu}^{U}(G^{U})^{\mu\nu}$ is invariant under chiral transformations .

In order to find an effective action for the colour chiral field U we study functional integrals

$$Z_{\psi}\left(G,A,RS\right)=\int D\overline{\psi}D\psi\exp i\int dxL_{\psi}\left(G,A\right)=\exp iW\left(G,A,RS\right)$$

$$Z_\psi\left(G^U,A^U,RS\right)=\int D\overline{\psi}D\psi\exp i\int dxL_\psi\left(G^U,A^U\right)=\exp iW\left(G^U,A^U,RS\right)\qquad(15)$$

which are also specified by a Regularization Scheme RS . They play the role of quantities Z and Z^U in definition of an effective action $W_{eff}(U)$. Thus, we obtain

$$W_{eff}(G,U,RS)=-i\ln\frac{Z_\psi\left(G,A,RS\right)}{Z_\psi\left(G^U,A^U,RS\right)}\qquad(16)$$

The usual way to calculate effective chiral action is to find an infinitesimal change $\delta_U W_{eff}$ (i.e.the anomaly) and integrate it up to U . We put $U=\exp\Theta$ and introduce the anomaly $A\left(x,\Theta\right)$

$$A\left(x,\Theta\right)=\frac{1}{i}\frac{\delta\ln Z_\psi\left(\exp\Theta\right)}{\delta\Theta}\qquad(17)$$

Then

$$W^\psi_{eff}\left(\Theta\right)=-\int d^4x\int_0^1 dsA\left(x;s\Theta\right)\Theta\left(x\right)=\int d^4xL^\psi_{eff}\left(U\right)-W_{WZW}\qquad(18)$$

where the Wess-Zumino-Witten term W_{WZW} describes topological properties of U and is represented by five-dimensional integral with $x_5=s$

$$W_{WZW}=\frac{i}{96\pi^2}\int d^5x\varepsilon_{\mu\nu\sigma\lambda\rho}tr[j^-_\mu L_{\nu\sigma}L_{\lambda\rho}+j^+_\mu R_{\nu\sigma}R_{\lambda\rho}+$$

$$\frac{1}{2}\left(j^-_\mu L_{\nu\sigma}U_sR_{\lambda\rho}U_s^{-1}+j^+_\mu R_{\nu\sigma}U_s^{-1}L_{\lambda\rho}U_s\right)-i\left(L_{\mu\nu}j^-_\sigma j^-_\lambda j^-_\rho+R_{\mu\nu}j^+_\sigma j^+_\lambda j^+_\rho\right)$$

$$-\frac{2}{5}j^-_\mu j^-_\nu j^-_\sigma j^-_\lambda j^-_\rho]\qquad(19)$$

where the following notations are used

$$j^-_\mu=\overline{D}_\mu U_sU_s^{-1},j^+_\mu=U_s^{-1}\overline{D}_\mu U_s,U_s=\exp s\Theta$$

$$\overline{D}_\mu U_s=\partial_\mu U_s+L_\mu U_s-U_sR_\mu$$

and the convention $\mu,\nu,...=1,2,3,4,5;L_5=R_5=0$ implied. We remind that $W_{WZW}=0$ for SU(2) gauge group.

Eliminating external colour axial fields, $A_\mu=0$, we get the effective chiral Lagrangian $L^\psi_{eff}\left(U\right)$ arising from integration over fermions with N_F flavors

$$L_{eff}\left(U\right)=N_Ftr_C\{\frac{f_0^2}{4}D_\mu UD^\mu U^{-1}$$

$$+\frac{1}{192\pi^2}\left[\frac{1}{2}\left[UD_\nu U^{-1},UD_\mu U^{-1}\right]^2-(UD_\nu U^{-1}UD^\nu U^{-1})^2\right]$$

$$+\frac{1}{96\pi^2}\left([UD^\mu U^{-1},UD^\nu U^{-1}](G_{\nu\mu}+UG_{\nu\mu}U^{-1})+G_{\mu\nu}UG^{\mu\nu}U^{-1}\right)\}-T,\qquad(20)$$

where the kinetic term contains a constant f_0^2 which is an analogue of the pion decay constant f_π^2 . The last term T contains higher derivatives and reflects presence of higher radial excitations,

$$T = \frac{N_F}{96\pi^2} tr_C D_\mu^2 U^{-1} D_\nu^2 U \tag{21}$$

This term should be taken into account with another degrees of freedom relevant to excited states and can be disregarded in the ground state problem [19].

Terms $d = 4$ do not depend on regularization scheme RS ; the constant f_0^2 may look different in different RS , but in applications it should be taken from phenomenology. It should be considered as a dimensional parameter.

The effective chiral Lagrangian $L_{QCD}(U)$ arising from the Yang-Mills lagrangians for G_μ and G_μ^U can be written as

$$L_{QCD}(U) = L_{YM}(G) - L_{YM}\left(G^U\right) =$$

$$= -\frac{1}{2g^2} tr_C \left[\frac{1}{2}\left(UG_{\mu\nu}U^{-1}G^{\mu\nu}\right) + \frac{1}{16}\left[UD_\mu U^{-1}, UD_\nu U^{-1}\right]^2\right] \tag{22}$$

It has the sign opposite to the sign at similar structures arising in bosonization.

The baryon number current B_μ is obtained by variation of the effective action $W_{eff}^\psi(U)$ due to the variation $G_\mu \to G_\mu + \omega_\mu$ with a color singlet ω_μ . Only the topological term W_{WZW} is involved thereby

$$B_\mu = -i\frac{\delta W_{eff}^\psi}{\delta\omega_\mu}$$

$$= \frac{N_F}{24\pi^2 N_C}\varepsilon_{\mu\nu\lambda\sigma} tr\left[UD^\nu U^{-1}UD^\lambda U^{-1}UD^\sigma U^{-1} - 3G^{\nu\lambda}\left(UD^\sigma U^{-1} - U^{-1}D^\sigma U\right)\right] \tag{23}$$

B_μ is normalized to give the baryon number $B = \frac{1}{3}$ for a quark.

0.3 Choice of background field. Action for soliton in vacuum

Colour configurations are always associated with background colour field G_μ because of necessity to maintain colour gauge invariance. In this respect, the case of colour solitons is quite different from the case of flavor solitons, where there is no flavour gauge invariance, and the external flavour gauge field can be eliminated from the chiral action. We consider the colour gauge group SU(2) with antihermitian generators $T_a = \frac{\tau_a}{2i}$, where τ_a are the Pauli matrices.

The background colour field should be chosen according to the problem under consideration. Our first step is to study a single colour soliton, i.e. a soliton in the vacuum of gluonic field. The gluonic vacuum Ψ_0 is characterized by the condensate

$$C_g = \left(\Psi_0, \frac{g^2}{4\pi^2}O_{\mu\nu}^a O^{\mu\nu a}\Psi_0\right) \cong \frac{g^2}{4\pi^2}G_{\mu\nu}^a G^{\mu\nu a} \neq 0 \tag{24}$$

that is by the non-zero vacuum expectation value of the Yang-Mills lagrangian for the full quantum field O_μ represented by the background vacuum field G_μ in our approximation. According to phenomenological descpription $C_g \succ 0$, so that G_μ is a chromomagnetic field in the real case of SU(3) gauge group. The vacuum field strength G_{kl} in the temporal gauge $G_0 = 0$ is constant up to a time independent gauge transformation. The effective Lagrangian L_{eff} is invariant under gauge transformations of background fields and the chiral field.

We shall consider the simplest case of a chromomagnetic vacuum background field , when it is an Abelian-type field which is a product a coordinate vector field V_k and a SU(2) color vector n^a

$$G_k^a = V_k n^a, V_k = -\frac{1}{2} V_{kl} x_l = -\frac{1}{2} \varepsilon_{klm} x_l \nu_m B, G_k = g G_k^a \frac{\tau_a}{2i} \tag{25}$$

where n^a is a constant unit vector in the colour space, ν_m is a constant unit vector in coordinate space, $\nu_m B = \frac{1}{2}\varepsilon_{mlk} V_{lk}$ is the vacuum chromomagnetism and B is related to the condensate $C_g = \frac{g^2}{2\pi^2} B^2$. In the vacuum all directions n^a and ν_l are equivalent, so that it is necessary to average over them at the end. Such a choice of vacuum field does not lead to stability troubles in QCD; although imaginary terms were detected at one-loop level [20], they disappear in all-loop treatment [21].

Let us write the chiral field in the usual way

$$U = \exp i \left(\frac{x_a \tau_a}{R} \right) F\left(R\right) = \cos F + i \mathbf{r} \sin F, r_a r_a = r^2 = 1, r_a \tau_a = \mathbf{r}, r_a = \frac{x_a}{R} \tag{26}$$

Under a gauge transformation $S(\overrightarrow{x})$ the chiral field U transforms together with the vacuum unit colour vector $\mathbf{n} = n_a \tau_a$ as

$$U' = S U S^+, \mathbf{n}' = S \mathbf{n} S^+ + S \partial_k S^+$$

and it is natural to restrict $S(\overrightarrow{x})$ by a condition $[S, U] = 0$.

Main structures in the Effective Chiral Lagrangian $L_{eff}\left(U\right)$ take on the following form

$$D_k U = \partial_k U + g \frac{V_k}{2i} \left[\mathbf{n}, i \mathbf{r} \right] \sin F$$

$$S_{kl} = \left[U D_k U^+, U D_l U^+ \right] =$$

$$= 4g \left(\left[\overrightarrow{\mathbf{n}}, \overrightarrow{\mathbf{r}} \right]_l V_k - \left[\overrightarrow{\mathbf{n}}, \overrightarrow{\mathbf{r}} \right]_k V_l \right) \frac{\sin^2 F}{R} + \partial_l U \partial_k U^+ - \partial_k U \partial_l U^+ \tag{27}$$

where $R^2 = x_k x_k$. The kinetic structure K and related non -Skyrme term N are given by

$$K = tr \left(D_l U^+ D_l U \right) = 2[(\partial_R F)^2 + \frac{2 \sin^2 F}{R^2}] + g^2 V^2 \left[\overrightarrow{n}, \overrightarrow{r} \right]^2 \sin^2 F \tag{28}$$

$$N = tr \left(D_l U^+ D_l U \right)^2$$

$$= 2\left((\partial_R F)^2 + \frac{2\sin^2 F}{R^2}\right)^2 + \frac{16}{15} g^4 V^4 \sin^4 F + \frac{8}{3} g^2 ((\partial_R F)^2 + 3\frac{\sin^2 F}{R^2}) V^2 \sin^2 F \qquad (29)$$

where $A^2 = V_k V_k$ and we have averaged over directions $\mathbf{n}$ in the SU(2) colour space putting $\overline{n_k} = 0, \overline{n_k^2} = \frac{1}{3}, \overline{n_k^4} = \frac{1}{5}, \overline{n_k^2 n_l^2} = \frac{1}{15}$.

Similarly, we average over directions $\overrightarrow{\nu}$ of field V_k in space of coordinates x_k and get

$$\overrightarrow{V} = \frac{1}{2}\left[\overrightarrow{\upsilon}, \overrightarrow{r}\right] RB, \overline{V^2} = \frac{1}{6} B^2 R^2, \overline{V^4} = \frac{1}{4}\left(1 - (\overrightarrow{r}, \overrightarrow{\nu})^2\right)^2_{av} B^4 R^4 = \frac{2}{15} B^4 R^4 \qquad (30)$$

It follows that the gauge field dependent part of the Skyrmion structure is given by

$$tr S_{kl} S_{kl} - tr(S_{kl} S_{kl})_{B=0} = \frac{32}{9} g^2 B^2 \sin^4 F \qquad (31)$$

while a mixed part of chirally transformed Lagrangian of the background vacuum field acquires $\sin^2 F$

$$tr G_{lk} U G_{lk} U^+ = -\frac{4}{3} g^2 B^2 \sin^2 F, G_{lk} = \frac{g}{2i} B \varepsilon_{lkt} \nu_t n, tr G_{lk} G_{lk} = -g^2 B^2 \qquad (32)$$

We are now able to write down the Effective Colour Static Lagrangian

$$L_{eff}(U, G_k) = -N_F \frac{f_0^2}{16\pi^2}[2((\partial_R F)^2 + 2\frac{\sin^2 F}{R^2}) + \frac{2}{9} g^2 B^2 R^2 \sin^2 F]$$

$$-\frac{N_F}{96\pi^2}[\left((\partial_R F)^2 + 2\frac{\sin^2 F}{R^2}\right)^2 + \frac{16}{225} g^4 B^4 R^4 \sin^4 F +$$

$$+\frac{2}{9} g^2 B^2 R^2 \left((\partial_R F)^2 + 3\frac{\sin^2 F}{R^2}\right) \sin^2 F] - (\frac{N_F}{48\pi^2} - \frac{1}{2g^2})\frac{2}{3} g^2 B^2 \sin^2 F$$

$$-\frac{1}{2\pi^2}\left(\frac{N_F}{12\pi^2} - \frac{1}{g^2}\right)\left[\frac{2\sin^2 F}{R^2}(\partial_R F)^2 + \frac{\sin^4 F}{R^4} + \frac{2}{9} g^2 B^2 \sin^4 F\right] \qquad (33)$$

where terms with gB arise from vacuum background field G_μ ,while $1/g^2$ terms come from the Yang-Mills lagrangian of G_μ .

The Euler-Lagrange equation for (33) for the soliton function $F(R)$ has the form

$$N_F f_0^2\left[\left(1 + \frac{g^2 B^2}{6} R^4\right)\sin 2F - 2R\partial_R F - R^2 \partial^2 F\right] +$$

$$+\left(\frac{N_F}{12\pi^2} - \frac{1}{g^2}\right)\left[\frac{\sin^2 F \sin 2F}{R^2} - \sin 2F (\partial_R F)^2 - 2\sin^2 F \partial^2 F + \frac{2}{3} g^2 B^2 R^2 \sin^2 F \sin 2F\right] +$$

$$+\frac{N_F}{24\pi^2}\left[\frac{2\sin^2 F \sin 2F}{R^2} - 2R(\partial_R F)^3 - 3R^2(\partial_R F)^2 \partial_R^2 F - 2\partial_R^2 F \sin^2 F - (\partial_R F)^2 \sin 2F\right] +$$

$$+\frac{N_F}{675}g^4B^4R^6\sin^2F\sin 2F$$
$$+\frac{N_F}{216\pi^2}g^2B^2R^4\left[\frac{3\sin^2F\sin 2F}{R^2}-4\partial_RF\frac{\sin^2F}{R}-\partial_R^2F\sin^2F\right]$$
$$-\frac{N_F}{432\pi^2}g^2B^2R^4(\partial_RF)^2\sin 2F+\frac{1}{3}\left(\frac{N_F}{24\pi^2}-\frac{1}{g^2}\right)g^2B^2R^2\sin 2F=0 \qquad (34)$$

This expression for the effective action contains terms $R^4\sin^4F$ and $R^2\sin^2F$ defining the asymptotic behaviour of $\sin F$ necessary to obtain finite static energy or mass

$$M=-4\pi\int dRR^2L_{eff}\left(U,G_k\right) \qquad (35)$$

The contribution to the mass functional M from the bosonized action sums from the kinetic term, $d=4$ terms and the contribution of the background vacuum field. It is easely to see that this part is positive definite and bounded from below and provides with the soliton configuration. The contribution from the Yang-Mills lagrangian (proportianal to $1/g^2$) is negative and can destabilize the soliton.

We introduce dimensionless variable $\rho=Ef_0R$, then the asymptotic behavior at large R of the decreasing function $F\left(\rho\right)$ is represented by the following equation

$$\partial_\rho\left[\rho^2\partial_\rho F\right]-2(1+C\rho^4)F=0, \qquad (36)$$

where the dimensionless parameter $C=\pi^2Cg/9(Ef_0)^4$ is related to the gluon condensate $Cg=g^2B^2/2\pi^2$. The solution of the Eq.(36) is modified Bessel functions of the second kind $K\left(\frac{3}{4},\sqrt{\frac{C}{2}}\rho^2\right)/\sqrt{\rho}$ and asymptotic behavior

$$F\rightarrow\rho^{-\frac{3}{2}}\exp\left(-\sqrt{\frac{C}{2}}\rho^2\right),\rho\rightarrow\infty \qquad (37)$$

which guarantees that the mass M is finite.

At small ρ, as it can be expected, the soliton function $F(\rho)$ behaves near the origin $\rho=0$ in the same manner as in the Skyrme model $F\left(\rho\right)\approx\pi-b\rho$.

Thus, the function F of the colour soliton is quite different from that of the flavor skyrmion.

It is easy to verify that the Baryon current B_μ is not influenced by the background vacuum field.

According to Witten analysis [3], statistics of the soliton is determined by the topological term W_{WZ} which is linear in time derivatives and reflects soliton behaviour under the rotation through a 2π angle. In the case of W_{WZ} built on chiral colour fields, statistics is defined by the sign $(-1)^{N_F}$, i.e. by odd or even number of flavors N_F . Baryon number B of soliton is equal to $B=N_F/N_C$. Thus, the simplest colour solitons will have $B=\frac{1}{3},\frac{2}{3}$ that corresponds to quark and diquark.

We consider a family of trial functions

$$F(\rho) = \pi \sqrt{\frac{1 - b\rho + a\rho^2}{1 + A\rho^5}} \exp(-\frac{A}{2}\rho^2) \tag{38}$$

where coefficients a and b are variational parameters and parameter $A = \sqrt{2\pi^2 Cg/9(Ef_0)^4}$. We also minimize the mass functional with respect to the scale transformation $\rho \to E\rho$. The functions (38) reflects the behaviour at the origin and large distances (37). We look for soliton configuration with $N_F = 1$. We use the value for the gluon condensate $C_g = (350MeV)^4$. For the case $\alpha_s = g^2/4\pi^2 \to \infty$ we find stable soliton solutions for the wide range of the unknown phenomenological parameter $f_0 = (10...60)MeV$. In the case of finite α_s we use the value for $f_0 = 16MeV$, which corresponds to the mass scale $\Lambda_C = 100MeV$ of the colour bosonization [7]. For the α_s we take the value when the corresponding terms in (33) change their signs to opposite: $M(\alpha_s \to \infty) = 460MeV$, $M(\alpha_s = 12\pi) = 362MeV$, $M(\alpha_s = 4\pi) = 120MeV$. For $\alpha_s = 3\pi$ the contribution from the last two terms in (33) is negative and its value exceeds all positive contributions to mass functional.

0.4 Conclusions and discussion

We have derived chiral colour action in background field. The action contains two parts: gauge invariant bosonization part and QCD part arising from the lagrangian of background field. The background field should satisfy standard conditions of QCD in background gauge. QCD part enters with the sign opposite to bosonization part. We applied this action to the case of soliton configuration defined as a configuration in a vacuum background field related to the gluon condensate, and considered the static case. Vacuum background field ensures exponential decrease in asymptotic region for chromomagnetic condensate, which is essential for stability of soliton and finiteness of mass. The (renorm-invariant) condensate is considered as a phenomenological quantity. Bosonization part of action does not depend explicitly on the coupling constant, while the QCD part contains it in denominator, because of renorm-invariance properties of the QCD background gauge. Negative contribution to mass from the QCD part increases as inverse coupling constant. Variational estimates with the trial function with proper asymptotics behaviour and the gluon condensate $(350Mev)^4$ shows that for the one flavor case the mass cannot be more then 460 MeV. Solitons definitely exist in bosonization part of action. Solitons exist in complete action, if one seriously considers perturbation results for the coupling constant. Solitons disappear when coupling constant becomes small, and the QCD part cancels stabilizing terms in the mass. This statement for the complete action is of indicative character, because of the simplest approximation used for the QCD part.

References

[1] T.H.R.Skyrme, Proc.R.Soc.London A260 (1961) 127

[2] A.P.Balachandran, V.P.Nair, S.G.Rajeev and A.Stern, Phys.Rev. D27 (1983) 1152, 2772

[3] E.Witten, Nucl.Phys.B223(1983) 422, 433

[4] N.S.Manton, Commun.Math.Phys. 111 (1987) 469

[5] L.D.Faddeev, Lett. Mat.Phys. 1 (1976) 289

[6] L.D.Faddeev and A.J.Niemi, Nature 387 (1997) 58; Phys.Lett. B449 (1999) 214; B464 (1999) 90

[7] A.A.Andrianov and Yu.V.Novozhilov, Phys.Lett. B 153 (1985) 422

[8] V.A.Andrianov and V.Yu. Novozhilov, Sov.J.Nucl.Phys. 43 (1986) 626

[9] D.B.Kaplan, Phys.Lett. B235 (1990) 163; Nucl.Phys. B 351 (1991) 357

[10] G. Gomelski, M.Karliner and S.B. Selipsky, Phys. Lett. B323 (1994) 182

[11] J.Ellis, Y.Frishman, A Hanany and M.Karliner, Nucl. Phys. B382 (1992) 189

[12] Y.Frishman. A.Hanany and M. Karliner, Nucl. Phys. B424 (1994) 3

[13] Y.Frishman, A.Hanany and M.Karliner, hep-ph/9507206

[14] A.Andrianov; Phys.Lett. 157B (1985) 425

[15] N.Karchev and A.Slavnov, Teor.Mat.Fys. 65 (1985) 192

[16] A.A.Andrianov, V.A.Andrianov, V.Yu.Novozhilov, Yu.V.Novozhilov Lett.Math.Phys. 11 (1986) 217

[17] R.D.Ball, Int.J.Mod.Phys.A, 5 (1990) 4391

[18] V.Yu.Novozhilov and Yu.V.Novozhilov, J.Chaos 12 (2001) 2789; quant-ph/9911108

[19] A.A.Andrianov, V.A.Andrianov, A.Manashov, Int.J.Mod.Phys. A6 (1991) 5435

[20] G.K.Savvidy, Phys.Lett. 71B (1977) 133

[21] E.Elizalde and J.Soto, Nucl.Phys. B283 (1987) 577